General Differential Geometry and General Lie Theory

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Abstract. — By "general" differential geometry and Lie theory we mean those parts of the theory which can be generalized to (almost) arbitrary base fields and -rings and to arbitrary dimension. In this Note we describe two basic features of such a theory: we interprete the tangent functor as the functor of scalar extension by dual numbers, and we investigate the structure of higher order tangent bundles from the point of view of the theory of connections. Full details are given in [1].

Géométrie différentielle et théorie de Lie générales

Résumé. — Par géométrie différentielle et théorie de Lie "générales" nous désignons les parties de la théorie que l'on peut étendre au cas de dimension arbitraire sur un corps (presque) arbitraire ou même sur des anneaux. Dans cette Note, nous décrivons deux aspects fondamentaux d'une telle théorie: nous interprétons le foncteur tangent comme un foncteur d'extension scalaire par des nombres duaux, et nous étudions la structure des fibrés tangents itérés sous le point de vue de la théorie des connexions. Une version détaillée est donnée dans [1].

Version française abrégé. – Dans [2], un calcul différentiel a été développé qui fonctionne sur tout anneau commutatif topologique $\mathbb{K}$ dont le groupe des éléments inversibles est dense dans $\mathbb{K}$, et les variétés et groupes de Lie modelés sur des $\mathbb{K}$-modules topologiques séparées ont été introduits.

Nous prouvons que, dans ce contexte, le fibré tangent TM d'une variété M sur $\mathbb{K}$ est, de manière naturelle, une variété définie sur l'anneau $\mathbb{K}[\varepsilon] = \mathbb{K}[x]/(x^2) \cong \mathbb{K} \oplus \varepsilon\mathbb{K}$ ($\varepsilon^2 = 0$) des *nombres duaux sur* $\mathbb{K}$. Ainsi le foncteur tangent peut être vu comme un foncteur d'extension scalaire de $\mathbb{K}$ à $\mathbb{K}[\varepsilon]$, ce qui donne une nouvelle et très simple approche aux "points infiniment voisins" d'André Weil [10].

Les fibrés tangents itérés $T^k M := T(T^{k-1}M)$ sont des variétés sur l'anneau $\mathbb{K}[\varepsilon_1]\ldots[\varepsilon_k]$; ils contiennent comme sous-fibrés des *fibrés de jets* $J^k M = (T^k M)^{\Sigma_k}$ (points fixes de l'action canonique du groupe symétrique Σ_k). Pour $k > 1$, ces fibrés, vus sur la base M, ne sont plus des fibrés vectoriels car leur fonctions de changement de carte sont multilinéaires et non linéaires en fibres (cf. formule (2) ci-dessous). Ceci nous amène à définir des *fibrés multilinéaires* comme étant des fibrés sur M dont les fonctions de transition sont multilinéares (dans un sens à préciser) en chaque fibre. Les fibres ne sont donc plus des espaces linéares (i.e. des $\mathbb{K}$-modules), mais plutôt des espaces sur lesquels une multitude de structures linéaires coexiste, permutées entre elles de manière simplement transitive par un groupe que nous appelons le *groupe multilinéaire spécial*.

En choisissant, de manière différentiable, dans chaque fibre une de ces structures linéaires, on obtient une *connexion multilinéaire* sur $T^k M$ (resp. sur un fibré multilinéaire quelconque). Pour $k = 2$ et $\mathbb{K} = \mathbb{R}$, ceci coïncide avec la notion usuelle de connexion affine sur TM. Partant d'une connexion affine L sur TM, nous construisons une suite de connexions multilinéaires $L^{k,\sigma}$ sur $T^{k+1}M$, paramétrée par $k \in \mathbb{N}$ et $\sigma \in \Sigma_k$. Un résultat principal affirme que la *courbure* de L peut être vue comme étant une "différence" entre $L^{2,\mathrm{id}}$ et $L^{2,(12)}$, et que les fibrés de jets $J^k M$ sont des sous-fibrés *linéares* par rapport aux $L^{k,\sigma}$.

Cette théorie s'applique aux cas des *groupes de Lie* G et des *espaces symétriques* M. Les fibrés tangents, $T^k G$, resp. $T^k M$, sont encore des groupes de Lie, resp. des espaces symétriques (sur $\mathbb{K}$

et même sur $\mathbb{K}[\varepsilon_1]\ldots[\varepsilon_k]$), et nous cherchons à analyser leur structure sur la base G, resp. sur M, en construisant d'abord des connexions canoniques qui correspondent à des trivialisations à gauche, resp. à droite. Dans le cas de caractéristique nulle, on peut construire, de manière purement algébrique, une *application exponentielle* pour les groupes $(T^kG)_e$, resp. $(J^kG)_e$, qui représente une connexion multilinéaire et totalement symétrique sur T^kG. Dans ce cas, on retrouve essentiellement la formule de Campbell-Hausdorff; dans le cas général, on trouve une formule similaire, mais beaucoup plus simple et qui est valable en toute caractéristique (formules (5) et (6)).

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1. Differential calculus. – We fix as base ring $\mathbb{K}$ a topological ring (commutative, with unit 1) such that the group $\mathbb{K}^\times$ of invertible elements is dense in $\mathbb{K}$. Following [2], we say that a continuous map $f : V \supset U \to W$ (where V, W are Hausdorff topological $\mathbb{K}$-modules, and U is open in V) is *differentiable* or *of class* C^1 if there exists a continuous map $f^{[1]} : U \times V \times \mathbb{K} \supset U^{[1]} \to W$ (where $U^{[1]} := \{(x,v,t) | \, x + tv \in U\}$, an open set in $V \times V \times \mathbb{K}$) such that, for all $(x,v,t) \in U^{[1]}$,

$$f(x+tv) - f(x) = t \cdot f^{[1]}(x,v,t). \tag{1}$$

By density of $\mathbb{K}^\times$ in $\mathbb{K}$, the map $f^{[1]}$ is then unique, and we may let $df(x) := f^{[1]}(x,v,0)$ and $Tf(x,v) := (f(x), df(x)v)$. We say that f is *of class* C^2 if it is of class C^1 and $f^{[1]}$ is of class $C^{[1]}$, and so on, thus defining classes C^k for all $k \in \mathbb{N} \cup \{\infty\}$. All basic rules of differential calculus, including Schwarz' lemma and a version of Taylor's formula, are very easily proved in this context (see [1] or [2]). However, in general it is impossible to integrate differential equations, not even the equation $df = 0$.

2. Smooth manifolds over general base fields and rings. – Smooth manifolds, modelled on a topological $\mathbb{K}$-module V, are defined in the usual way using charts and atlasses (not necessarily maximal ones), and similarly for vector bundles and more general bundles. The tangent bundle TM of a manifold M is defined via its transition maps Tf, if f is a transition map of M, and not via point derivations. *Vector fields* are smooth sections of the tangent bundle, and as usual they form a Lie algebra under their Lie bracket (see [1] or [2]).

3. Tangent functor and scalar extensions. – The *ring of dual numbers over* $\mathbb{K}$ is the ring $\mathbb{K}[\varepsilon] := \mathbb{K}[x]/(x^2) \cong \mathbb{K} \oplus \varepsilon\mathbb{K}$ with $\varepsilon^2 = 0$. We call it also the *tangent ring*, because of the following

Lemma 1. – *The tangent maps of the ring operations on $\mathbb{K}$ turn the tangent bundle $T\mathbb{K}$ into a ring which is naturally isomorphic to $\mathbb{K}[\varepsilon]$. The ring $T\mathbb{K}$ has again a dense unit group.*

A central result of our theory generalizes this fact to the whole category of manifolds:

Theorem 1. – *If M is a smooth manifold over $\mathbb{K}$, then TM is, in a natural way, a manifold over the tangent ring $T\mathbb{K} \cong \mathbb{K}[\varepsilon]$, and if $f : M \to N$ is smooth over $\mathbb{K}$, then $Tf : TM \to TN$ is smooth over $T\mathbb{K}$. Summing up, the tangent functor T can be seen as a functor of scalar extension from $\mathbb{K}$ to $T\mathbb{K}$ in the category of smooth manifolds.*

The theorem is proved by writing Equation (1) as a commutative diagram and realizing that, applying the tangent functor to this diagram, via Lemma 1 we get the same kind of diagram for $\mathbb{K}$ replaced by $T\mathbb{K}$. In a way, this result contains a new justification of "infinitesimals" as proposed by A. Weil in [10], having the advantage not to use the heavy logical or model-theoretic machinery introduced in synthetic differential geometry, cf. e.g. [8].

Corollary 1. – *The higher order tangent bundles $T^kM := T(T^{k-1}M)$ are smooth manifolds over the ring $T^k\mathbb{K} \cong \mathbb{K}[\varepsilon_1, \ldots, \varepsilon_k]$, where $\varepsilon_i^2 = 0$, $\varepsilon_i\varepsilon_j = \varepsilon_j\varepsilon_i$.*

4. Jet functor and scalar extensions. – The symmetric group Σ_k acts in a natural way by bundle automorphisms on the higher order tangent bundle $T^k M$ over M. It is easily proved that the fixed space, $J^k M := (T^k M)^{\Sigma_k}$, is a subbundle of $T^k M$.

Corollary 2. – *The bundle $J^k M$ is a smooth manifold over the ring $J^k \mathbb{K} := (T^k \mathbb{K})^{\Sigma_k} \cong \delta^{(1)}\mathbb{K} \oplus \ldots \oplus \delta^{(k)}\mathbb{K}$, where $\delta^{(j)} := \sum_{i_1 < \ldots < i_j} \varepsilon_{i_1} \cdots \varepsilon_{i_j}$. If the integers $2, \ldots, k$ are invertible in $\mathbb{K}$, then $J^k \mathbb{K}$ is isomorphic to a truncated polynomial ring $\mathbb{K}[x]/(x^{k+1})$.*

We call J^k the *k-th order jet functor* since, in usual differential geometry, $J^k M$ corresponds to the bundle of k-jets of curves in M; our definition has advantages in the case of positive characteristic since in this case it is not always possible to "integrate" jets to suitable (locally defined) curves.

5. Multilinear bundles. – Our approach to the theory of connections is by analyzing the structure of the bundles $T^k M$ (resp. of the bundles $T^{k-1}F$ for a general vector bundle F over M) *over the base* M. For $k > 1$, these bundles are no longer vector bundles, but still their fibers have a rather rich algebraic structure: they are *multilinear spaces*, and thus $T^k M$ and $T^{k-1}F$ are *multilinear bundles over* M. In order to explain these concepts, let us start with the transition functions of the bundle $T^k M$ which are of the form $T^k f$, where f is a typical transition function of M. Using multi-indices $\alpha \in I_k := \{0,1\}^k$, a typical point $u \in T^k M$ is represented in a bundle chart by $u = x + \sum_{\substack{\alpha \in I_k \\ \alpha \neq 0}} \varepsilon^\alpha v_\alpha$, with $\varepsilon^\alpha = \prod_i \varepsilon_i^{\alpha_i}$ and v_α belonging to the model space V of the manifold M. Then we have

$$T^k f(x + \sum_{\substack{\alpha \in I_k \\ \alpha \neq 0}} \varepsilon^\alpha v_\alpha) = f(x) + \sum_{j=1}^{k} \sum_{\substack{\alpha \in I_k \\ |\alpha| = j}} \varepsilon^\alpha (\sum_{l=1}^{j} \sum_{\Lambda \in \mathcal{P}_l(\alpha)} d^l f(x)(v_{\Lambda^1}, \ldots, v_{\Lambda^l})), \tag{2}$$

where $|\alpha| := \sum_i \alpha_i$, and Λ runs through the set $\mathcal{P}_l(\alpha)$ of *partitions of α of length l*, i.e. $\Lambda = (\Lambda^1, \ldots, \Lambda^l)$ with $\Lambda^i \in I_k$ such that $\sum_i \Lambda^i = \alpha$ and $\Lambda^1 < \ldots < \Lambda^l$ with respect to some fixed total ordering of I_k (lexicographic, for instance). The formula suggests to consider self-maps $g := g_b : E \to E$ of the fiber $E := (T^k M)_x$ over $x \in M$ of the form

$$g(x + \sum_{\substack{\alpha \in I_k \\ \alpha > 0}} \varepsilon^\alpha v_\alpha) = x + \sum_{\substack{\alpha \in I_k \\ \alpha > 0}} \varepsilon^\alpha v_\alpha + \sum_{j=2}^{k} \sum_{\substack{\alpha \in I_k \\ |\alpha| = j}} \varepsilon^\alpha (\sum_{l=2}^{j} \sum_{\Lambda \in \mathcal{P}_l(\alpha)} b^\Lambda (v_{\Lambda^1}, \ldots, v_{\Lambda^l})), \tag{3}$$

where $b = (b^\Lambda)_\Lambda$ is a family of $\mathbb{K}$-multilinear maps $b^\Lambda : V^l \to V$ (not necessarily symmetric, and depending on x), indexed by all non-trivial partitions Λ of elements of I_k. Putting this into a purely algebraic context, we define a *multilinear space* to be given by a collection of $\mathbb{K}$-modules V_α, $\alpha \in I_k$, together with the (non-linear) action of the *special multilinear group* $\mathrm{Gm}^{1,k}(E)$ on the *total space* $E := \oplus_{\substack{\alpha \in I_k \\ \alpha > 0}} V_\alpha$, where $\mathrm{Gm}^{1,k}(E)$ is given by all maps of the form (3) when $b = (b^\Lambda)_\Lambda$ runs over all families whith $\mathbb{K}$-multilinear maps $b^\Lambda : V_{\Lambda^1} \times \ldots \times V_{\Lambda^l} \to V_\alpha$ ($\alpha = \sum_i \Lambda^i$). The action of the group $\mathrm{Gm}^{1,k}(E)$ on the fibers of $T^k M$, resp. of $T^{k-1}F$, is an intrinsic feature, and hence these bundles are *multilinear bundles over* M in the sense that their fibers carry a canonical structure of a multilinear space. For $k = 2$ all formulas simplify considerably; this is related to the fact that in this (and only in this) case the group $\mathrm{Gm}^{1,2}(E)$ is abelian.

6. Linear connections. – A *linear structure* on a fiber bundle F over M is given by fixing, in a smooth way, on each fiber F_x a linear (i.e. $\mathbb{K}$-module) structure. A *linear connection* on a vector bundle F over M is given by fixing a linear structure L on TF over M which, on every fiber E, is conjugate under the group $\mathrm{Gm}^{1,2}(E)$ to one (and hence to all) linear structures induced on E by bundle charts of TF (i.e. charts coming from charts of F). It can be shown that, in all classical situations, this definition coincides with more traditional ones (e.g., via connection one-forms, connectors or covariant derivatives); in our general situation this definition seems to have most advantages.

7. Multilinear connections on multilinear bundles. – Generalizing the preceding definition, we say that a *multilinear connection on the multilinear bundle* T^kF is given by fixing a linear structure L on T^kF over M which, on every fiber E, is conjugate under the group $\mathrm{Gm}^{1,k+1}(E)$ to one (and hence to all) linear structures induced on E by bundle charts of T^kF. Such connections are (in the classical situation) special cases of Ehresmann-connections; this follows from the following

Theorem 2. *A multilinear connection L on T^kM induces an isomorphism of bundles over M,*

$$\Phi : \bigoplus_{\substack{\alpha \in I_k \\ \alpha > 0}} \varepsilon^\alpha TM \to T^kM, \tag{4}$$

where $\varepsilon^\alpha TM$ is just a formal copy of TM, and the direct sum is a direct sum of bundles over M. A similar statement holds for multilinear connections on $T^{k-1}F$.

For $k = 2$ and $\mathbb{K} = \mathbb{R}$, the preceding result is known as the Dombrowski Splitting Theorem (cf. [4, Theorem X.4.3]).

8. Curvature. – Our main strategy of constructing multilinear connections is by "deriving linear connections"; this operation is canonical up to the chosen order $(\varepsilon_1, \ldots, \varepsilon_k)$ when interpreting the tangent functor as scalar extension by $\mathbb{K}[\varepsilon_i]$; the "difference" between linear structures belonging to different orders gives rise to curvature:

Theorem 3.

(1) *Given linear connections L on F and L' on TM, one can construct, in a canonical way, a sequence of "derived linear structures" $L^{k,\sigma}$ on T^kF, indexed by $k \in \mathbb{N}$ and $\sigma \in \Sigma_k$.*

(2) *If L' is torsionfree, then $L^{2,\mathrm{id}}$ and $L^{2,(12)}$ are related via a well-defined fiberwise trilinear map which does not depend on L', called the* curvature *of L.*

(3) *The subbundles $J^kF \subset T^kF$ are* linear *subbundles with respect to the linear structures $L^{k,\sigma}$, and the linear structure induced on J^kF depends only on k and not on σ.*

It can be shown that the interpretation of curvature from (2) coincides with all classical ones. Moreover, there is a conceptual interpretation of *Bianchi's identities* in this context.

9. Lie groups and symmetric spaces. – *Lie groups* over $\mathbb{K}$ are defined to be groups in the category of manifolds over $\mathbb{K}$, and *symmetric spaces* over $\mathbb{K}$ are defined to be manifolds with a smooth multiplication map $\mu : M \times M \to M$, $(x, y) \mapsto \mu(x, y) = \sigma_x(y)$ satisfying a version of the axioms given by O. Loos in [7] (here we need that $\frac{1}{2} \in \mathbb{K}$, see also [3]). The *Lie functor* assigning to a Lie group a Lie algebra and to a symmetric space with base point a Lie triple system can be defined as in the classical situation (cf. [1], [2], [3]).

Theorem 4. *Assume G is a Lie group over $\mathbb{K}$, resp. M a symmetric space over $\mathbb{K}$.*

(1) *T^kG and J^kG are Lie groups over $T^k\mathbb{K}$, resp. over $J^k\mathbb{K}$, and similarly for T^kM and J^kM.*

(2) *Left- and right-trivialization of TTG induce two canonical linear connections L^l and L^r on TG.*

(3) *The fibers of TTM over M are abelian symmetic spaces whose symmetric space structure is induced by a unique linear structure L^S on TTM; this structure defines a canonical torsionfree linear connection on TM.*

(4) *If $M = G$ is a Lie group, considered as symmetric space, then $L^S = \frac{L^l + L^r}{2}$.*

The group T^kG can be trivialized from the left and from the right, and one can show that this coincides with the trivializing map Φ coming from the right-, resp. from the left connection, L^r resp. L^l. Then a combinatorial formula for the group structure of $(T^kG)_e$ (and hence also of $(J^kG)_e$) in terms of the Lie bracket can be given:

Theorem 5. *With respect to the identification* $(T^kG)_e \cong \oplus_{\substack{\alpha\in I_k \\ \alpha>0}} \varepsilon^\alpha \mathfrak{g}$ *given by the left trivialization, the group structure of* $(T^kG)_e$ *is described by the product*

$$\sum_{\alpha>0} \varepsilon^\alpha v_\alpha * \sum_{\beta>0} \varepsilon^\beta w_\beta = \sum_{\gamma>0} \varepsilon^\gamma z_\gamma$$

with

$$z_\gamma = v_\gamma + w_\gamma + \sum_{j=2}^{k} \sum_{\Lambda\in\mathcal{P}_j(\gamma)} [\ldots[[v_{\Lambda^j}, w_{\Lambda^1}], w_{\Lambda^2}], \ldots, w_{\Lambda^{j-1}}], \tag{5}$$

and inversion is given by

$$(\sum_{\alpha>0} \varepsilon^\alpha v_\alpha)^{-1} = \sum_{\gamma>0} \varepsilon^\gamma (-v_\gamma + \sum_{j=2}^{k} (-1)^j \sum_{\Lambda\in\mathcal{P}_j(\gamma)} [v_{\Lambda^j}, [v_{\Lambda^{j-1}}, \ldots [v_{\Lambda^2}, v_{\Lambda^1}]]]). \tag{6}$$

There are similar formulae with respect to the right trivialization. By restriction to Σ_k-invariants, one gets the corresponding multiplication and inversion formulae for jets, i.e. for the group $(J^kG)_e$.

Formula (5) may be seen as an analog of the Campbell-Hausdorff formula; it is much simpler and works in arbitrary characteristic, but nevertheless the combinatorial structure of its restriction to Σ_k-invariants is fairly complicated. Similar formulae can be given for symmetric spaces with respect to the linearization coming from the connection L^S.

10. Exponential map and exponential jet. – Formulae (5) and (6) show that $(T^kG)_e$, with respect to left- or right-trivialization, is a *polynomial group of degree at most k* which we define, in a purely algebraic context, as a $\mathbb{K}$-module together with a group structure such that product and inversion are polynomial maps and the degree of the iterated product maps $m^{(j)}(x_1, \ldots, x_j) = x_1 \cdots x_j$ is, for all $j \in \mathbb{N}$, bounded by k.

Theorem 6. *Assume M is a polynomial group over $\mathbb{K}$ and that the integers are invertible in $\mathbb{K}$. Then there exist polynomials* $\exp : M \to M$ *and* $\log : M \to M$*, inverse of each other and of degree at most k, uniquely characterized by the properties:*
(1) *the term of degree one of* $\exp$ *is the identity map of M,*
(2) *for all $n \in \mathbb{Z}$ and $X \in M$, we have* $\exp(nX) = (\exp X)^n$.
The polynomials $\exp$ *and* $\log$ *can be described by explicit formulas in terms of $m^{(j)}$, $j = 1, \ldots, k$.*

Our proof of this result is much more elementary than the ones of corresponding results in the theory of *formal groups* (cf. [9]). Applying Theorem 6 to the polynomial group $M = (T^kG)_e$, we get a canonical exponential map

$$\exp_k : \oplus_{\substack{\alpha\in I_k \\ \alpha>0}} \varepsilon^\alpha \mathfrak{g} \to (T^kG)_e$$

with inverse $\log_k$. This map contains all information of the k-th order Taylor expansion of a "usual" exponential map of the group G (if it exists !) The group structure of $(T^kG)_e$ with respect to $\exp_k$ is described by the k-th order Taylor polynomial of the Campbell-Hausdorff formula. Taking the projective limit $k \to \infty$, we get a formal power series group, together with its "exponential jet" $\exp_\infty$, which in the classical, real or p-adic, theory ([9]) corresponds to the germ of G. Finally, similar results, including the existence of an exponential jet, can be proved for symmetric spaces.

11. Diffeomorphism groups. Using their theory of *Weil functors*, G. Kainz and P. Michor have shown that the space of sections of bundles such as T^kM and J^kM (for M finite dimensional over $\mathbb{K} = \mathbb{R}$) carry a canonical group structure (see [4], [5]). In our context, this is generalized and interpreted in terms of the groups $\mathrm{Diff}_{T^k\mathbb{K}}(T^kM)$ of all diffeomorphisms of T^kM that are smooth over the ring $T^k\mathbb{K}$ (recall from Cor. 1 that T^kM is a manifold over this ring).

Theorem 7.
(1) *The space $\mathfrak{X}^k(M)$ of smooth sections of the bundle T^kM over M carries a canonical group structure which turns it into a polynomial group (in the sense defined above). The group structure is defined by the property that every section $X : M \to T^kM$ admits a unique extension to a $T^k\mathbb{K}$-smooth diffeomorphism $\widetilde{X} : T^kM \to T^kM$.*
(2) *The group $\mathrm{Diff}_{T^k\mathbb{K}}(T^kM)$ is a semidirect product of the group $\mathrm{Diff}_{\mathbb{K}}(M)$, which is imbedded via $f \mapsto T^kf$, and the (normal) polynomial group $\mathfrak{X}^k(M)$, which is imbedded via $X \mapsto \widetilde{X}$.*

Theorem 7 can be interpreted by saying that, in may respects, the group $G = \mathrm{Diff}_{\mathbb{K}}(M)$ behaves like a Lie group with higher order tangent groups $T^kG = \mathrm{Diff}_{T^k\mathbb{K}}(T^kM)$. For instance, the structure formulas from Theorem 5 apply, and in case of characteristic zero one may define an exponential mapping $\exp_k$ which should be seen as a partial analog of the map associating to a vector field its flow at time $t = 1$.

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